ch-3

Conicoids

Classification of conicoids.

These are classified into 6 types :

Name of Surface	Standard form
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of two sheets	$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Elliptic paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}$
Hyperbolic paraboloid	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$
Elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$

Ellipsoid

Trace the locus of the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ — ①

where a, b, c are positive and $a>b>c$

→ **Axes Intersection**

The surface ① meets x-axis

where $y=0$, $z=0$

from ① $\frac{x^2}{a^2}=1$

$x^2=a^2$

$x=\pm a$

points of Intersection of x-axis $(a,0,0)$ $(-a,0,0)$

points of Intersection of y-axis $(0,b,0)$ $(0,-b,0)$

points of Intersection of z-axis $(0,0,c)$ $(0,0,-c)$

Traces on the coordinate planes :-

The surface meets the yz-plane where, putting $x=0$ in ①

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

It is an ellipse whose centre is the origin and major axis $= 2b$ lies along the y-axis and minor axes $= 2c$ lies along the z-axis.

IIIly, it meets the zx-plane in the ellipse $\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$ whose centre is origin, major axis $2a$ lies along x-axis, minor axis $= 2c$ lies along z-axis

Also, it meets the xy-plane in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose centre is the origin, major axis $= 2a$ lies along x-axis and minor axis $= 2b$ lies along the y-axis.

* Generated by a variable curve

The surface meets the plane $z = k$ where

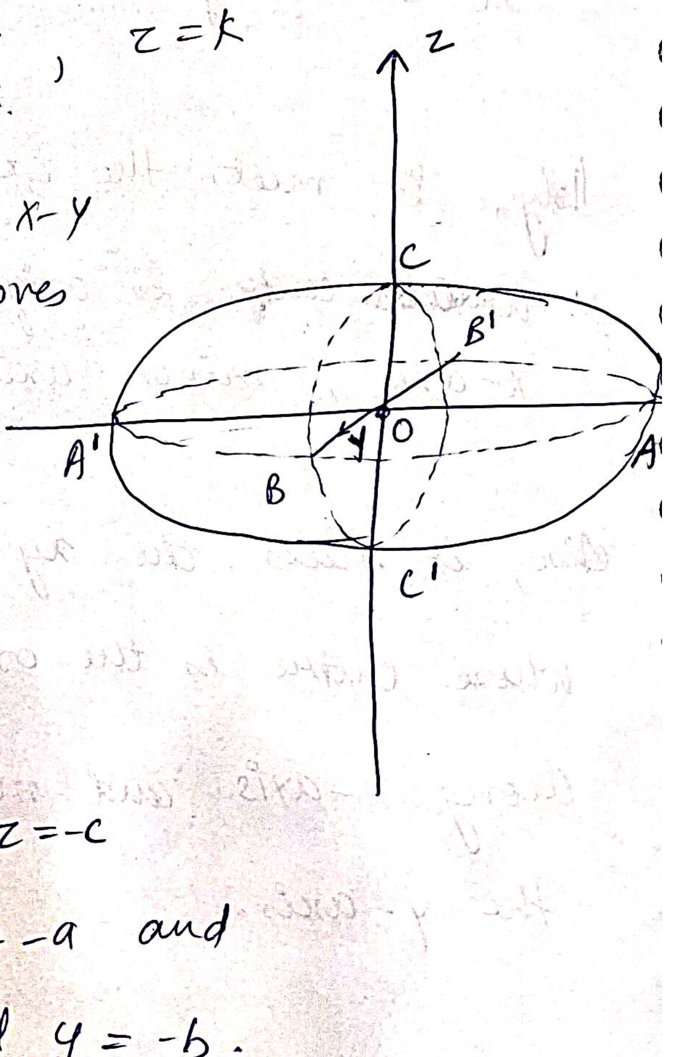
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{k^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{k^2}{c^2}$$

$\therefore$ The surface is generated by a variable ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{k^2}{c^2}, \quad z = k$$

whose plane is parallel to x-y plane and centre $(0,0,k)$ moves on z-axis.



The ellipse ② is real only if

$$1 - \frac{k^2}{c^2} > 0$$

$k^2 < c^2$ i.e $|k| < |c|$

i.e k lies lies b/w $z = c$ and $z = -c$

IIly it lies b/w $x = a$ and $x = -a$ and b/w the planes $y = b$ and $y = -b$.

$\therefore$ the surface is closed.

Hence the shape of the surface is shown in figure.